Prediction and Mitigation of Start-up Torsional Vibration Issues in Very High Load-Inertia Systems using Transient Torsional Analysis

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Abstract

Mechanical systems which include rotating and/or reciprocating components can experience vibration problems during operation. It is important that the system be fully analyzed in the design stage to avoid costly field modifications or equipment failures. Torsional vibration analysis is a critical analysis component of any rotating or reciprocating machinery system.

In most cases, a torsional vibration analysis focuses on the steady-state operation of the system. The rotational inertia of the driver equipment in the system is generally much larger than that of the load, under which conditions start-up from rest occurs in a matter of seconds. Because of this, events during transience are not considered to be a driving factor in the system design, and transient torsional analyses are rarely performed.

For systems where the inertia of the load is significantly greater than the inertia of the driver, the transient period tends to be much longer. For the case of electric drivers, this results in a considerable amount of revolutions being spent on the unstable part of the torque vs. RPM curve. This, together with the high torque needed to accelerate the large load inertia, can cause high amplitude torsional vibrations during start-up. These vibrations can cause high torques and stresses in the system during start-up, and may be sufficient to cause catastrophic failure of the system. Thus, it is critical to consider the start-up transient period when selecting system components, since choosing improper parameters could result in damage to the system during start-up.

This paper presents the results of a case study focusing on issues which may occur in a very high-inertia load torsional system. For the scope of this paper, we nominally define a "very high load inertia" as one that is more than 3-5 times greater than the total driver inertia. It also provides a general investigation into the effectiveness of various methods of mitigating the effect of these torsional issues during start-up.

1. Introduction

A torsional vibration analysis is an important component to include in a full analysis study on rotating or reciprocating machinery. These systems generally consist of a driver, commonly an electric induction motor or a gas engine, connected to a load such as a compressor, pump, or fan, by a coupling.

In most cases, the rotary inertia of the driver is several times greater than that of the load, so the system will reach its steady-state operating speed in a few seconds. Therefore, a steady-state torsional analysis, which is performed for the various run conditions of the system, is usually sufficient to cover the worst case operating scenarios of the system. If the torques and stresses during start-up are slightly higher than at steady-state, it is assumed that the total number of cycles reached during start-ups over the system's operating lifetime will be low enough so as not to approach the fatigue life limit of the components.

However, for systems where the rotary inertia of the load is very high compared to that of the driver, it is important to consider the performance of the system during start-up, as the period of transience before the system reaches steady-state is much longer than for systems where the load inertia is low. Problems during start-up may also be exacerbated by certain characteristics of the system such as mode of operation, equipment sizing, and coupling selection.

This paper presents findings based on a case study of a transient torsional analysis, and proposes some guidelines for the design and operation of rotational machinery systems with very high inertia loads through the analysis of the system's transient behaviour during start-up. The case study results also demonstrate the possible ramifications of failing to consider the start-up performance of such systems.

Most of the existing literature on the prediction and mitigation of torsional vibrations in rotating equipment focuses on steady-state vibrations. A thorough paper by Corbo [1] on designing against torsional vibrations only briefly touches on the considerations for predicting torsional vibrations during transience. This paper suggests the use of modal or Fourier transform methods to predict transient vibrations.

Papers by Wang [2] and Galloway [3] demonstrate the formulation of systems of simplified equations of motion, based on transforms and amplification factors, to predict the motion of simple torsional systems under various loads. Another publication by Szenasi and von Nimitz [4] shows the governing time-transient based on first principles. However, the results and discussion in the paper focus on reducing vibrations through modification of the system's torsional natural frequencies, and provides few comparative results of the system performance.

These three papers were written in the 1970s, at a time in which the computing power to be able to perform a thorough time-transient analysis using a differential equation formulation was onerous. Instead, they used less computer-intensive methods such as calculating dynamic amplification factors. In this paper, we will use a full time-domain differential equation analysis to predict the torsional behaviour of a system during various start-up conditions, and we will investigate the effects that changing the system's geometric and operating parameters have on the torsional vibration levels during start-up.

The paper is organized as follows: In Section 2, the modeling methodology for a general torsional system is presented, including assembly of the lumped-parameter geometric model, and the set-up of the mathematical equations and numerical solver required for a transient torsional analysis. Section 3 covers the external supply and load torques which are applied to the system, and how they drive the mechanism of transient behaviour in the system. In Section 4, a case study is presented of a transient torsional analysis which was performed on a system with a very high-inertia load. The results presented here serve as a base case for the proposed solutions which are discussed in Section 5. Although the methods of mitigating torsional vibrations which are presented are applied to the case study system, they are meant to provide general guidelines when designing against torsional vibration issues during start-up. In this paper, we aim to demonstrate the value of analyzing the transient torsional behaviour of certain systems.

2. Torsional System Model

The construction of a torsional model from the system's geometric parameters is the first step in the analysis. A mathematical system of equations based on the equations of motion of the system is also required to predict the behaviour of the system.

2.1 Lumped-parameter Model

For a torsional vibration analysis, the drive train system is generally modeled as a series of discrete rotary inertias, representing the masses and inertias of the system, connected by discrete torsional springs, representing the stiffnesses of shafts and other mechanical connections.

For the purposes of this paper, we will consider the particular case of a torsional system where a load is driven by an electric induction motor.

For torsional vibration analyses, electric motors can be modeled as a single lumped inertial mass since most of their inertia is concentrated at the rotor core in the magnetic bars and windings as a fairly stiff assembly. For simplicity of our analysis, we make the assumption that the load in the system is one that can also be reasonably approximated as a single inertia. Examples of such loads include vane, screw, and certain types of centrifugal compressors, as well as fans and turbines.

The coupling is modeled as two rotary inertias, connected by a spring with the torsional stiffness of the coupling. Since the coupling is a geometrically complex part, its inertia and stiffness properties are tested and reported by the manufacturer.

The motor inertia is connected to the coupling inertia by a spring with the torsional stiffness of the motor shaft, and the load inertia is connected to the other coupling inertia by a torsional spring representing the stiffness of the load shaft.

The schematic of the lumped-parameter torsional model is shown in Figure 1. The inertias and stiffnesses in the torsional model will be based on the system's properties as listed in Table 1.

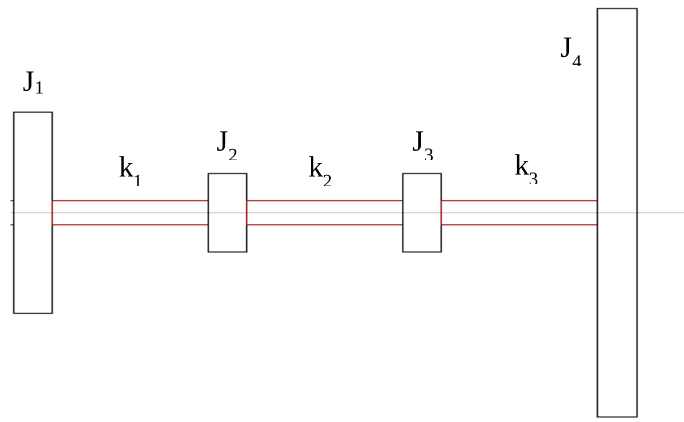

Figure 1: Torsional Geometric Model

Table 1: Torsional Model Parameters

Rotary Inertia	**Torsional Stiffness**
Motor Inertia (J_1)	Motor Shaft Stiffness (k_1)
½ Coupling Inertia (J_2)	Coupling Stiffness (k_2)
½ Coupling Inertia (J_3)	Load Shaft Stiffness (k_3)
Load Inertia (J4)	

The motor and load shaft stiffnesses are defined by the portions of the shaft between the coupling connection point and the center of the load inertia, since these are the sections of the shaft that are torsionally loaded. Parts of the shaft between the load inertia and the non-drive end of the shaft are ignored in the stiffness calculation, since they are not under any torsional load.

Drive shafts for motors, fans, and other rotary machines are often comprised of a series of stepped circular shaft portions, the largest of which is at the center, and the smallest of which is at the drive end. To calculate the overall equivalent stiffness, k_{eq}, the stiffness of each individual portion is combined in parallel [5]. The equations for this calculation are shown:

$$k_{shaft} = \frac{JG}{L} = \frac{\pi}{32} d^4 \frac{G}{L} \tag{1}$$

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_n} \tag{2}$$

In Eq. (1), J is the polar moment of inertia for a circular shaft, d is the shaft diameter, G is the shear modulus of the shaft material (around 80 GPa for most steels), and L is the length of the shaft portion.

2.2 Mathematical Model

The mathematical formulation of the torsional analysis is based on application of Newton's Second Law in a rotational sense, that is, by summing torques/moments and externally applied torsional forces. This general formulation is applied to each inertial mass as follows.

$$J_i\ddot{\theta}_i = k_{i-1}(\theta_{i-1} - \theta_i) + k_i(\theta_{i+1} - \theta_i) + \Sigma T_{ext,i} \tag{3}$$

Note that this formulation assumes that no damping is present in the system. Without changing the generality of the methodology to follow, damping with respect to ground (i.e., that afforded by bearings), as well as internal damping, can easily be added as long as they are proportional to the inertial velocities ($\dot{\theta}_i$).

The inertias at each extreme end of the system will have only one spring force acting on them, rather than the two shown in the general form of the torque balance in Eq. (3). For this torsional system, an external positive supply torque will be applied to the motor inertia, and an external negative load torque will be applied to the load inertia. The other system inertias will have zero external torque applied to them.

The system of mathematical equations for the torsional model as given in Table 1 will consist of four second-order ordinary differential equations. The state variables will be the angular deflections (θ_i) and angular velocities ($\dot{\theta}_i$ or ω_i) at each rotary inertia.

The angular deflections and velocities will be functions of time. The external torques, in this case, are functions of the rotational speed. These torques are defined by means of "torque-speed curves", which are provided for driver and load equipment, and are important to consider when studying the system's transient response. These system curves are discussed in further detail in section 3.

The time-domain solution to the system of equations can be solved using a variety of numerical methods for systems of linear ordinary differential equations. Some of these methods include Euler's forward and backward methods, and a variety of Runge-Kutta methods. All mathematical simulations in this paper are performed using the classic fourth-order Runge-Kutta method.

The solution to this system of equations will consist of the angular deflections and velocities of each inertia as functions of time. Individual spring torques in the system are important as indicators of system performance, and they are defined by the relative angular deflection between two inertias, acting across a shaft portion or component represented by a torsional spring. The torques across the "i^{th}" spring in the system can be calculated using a modified version of Hooke's Law:

$$T_i = k_i(\theta_{i+1} - \theta_i) \tag{4}$$

The stresses in the shafts and other components can be calculated using the known torques. For a torsionally loaded circular shaft, the maximum shear stress in the shaft occurs at the outer surface, and is calculated as:

$$\tau_{max} = \frac{Tc}{J} = \frac{T\left(\frac{d}{2}\right)}{\frac{\pi}{32}d^4} = \frac{T}{\frac{\pi}{16}d^3} \tag{5}$$

Assessment of torques and stresses on various system components is a critical part of a torsional analysis, as it provides a means of calculating the operating life of the system under the given operating loads.

2.3 System Eigenvalues

The eigenvalues of the system of equations represent the torsional natural frequencies of the system. In matrix form, the system of ordinary differential equations can be arranged as follows for a system of 4 inertias connected in series by torsional springs:

$$\begin{bmatrix} J_1 & 0 & 0 & 0 \\ 0 & J_2 & 0 & 0 \\ 0 & 0 & J_3 & 0 \\ 0 & 0 & 0 & J_4 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \\ \ddot{\theta}_4 \end{bmatrix} + \begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & k_1 + k_2 & -k_2 & 0 \\ 0 & -k_2 & k_2 + k_3 & -k_3 \\ 0 & 0 & -k_3 & k_3 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} \sum T_1 \\ \sum T_2 \\ \sum T_3 \\ \sum T_4 \end{bmatrix} \tag{6}$$

In a simplified form, these matrix equations become:

$$[J][\ddot{\theta}] + [K][\theta] = [T] \tag{7}$$

The eigenvalues are related to the system's free response, that is, where all external torques in the matrix T are identically zero. Assuming a sinusoidal excitation, for which $\ddot{\theta} = -\omega^2\theta$, the matrix equation in (7) can be rearranged as:

$$\{[K][J]^{-1} - \omega^2[I]\}[\theta] = 0 \tag{8}$$

Comparing this equation to the general form of an eigenvalue problem as shown in Eq. (9), we can see that the eigenvalues of the matrix $[K][J]^{-1}$ are the eigenvalues of the system, ω^2.

$$\{[A] - \lambda[I]\}[X] = 0 \tag{9}$$

When the square roots of these eigenvalues are taken, we obtain the system's torsional natural frequencies. Computation of these natural frequencies will be important when analyzing the system's transient response in later sections.

3. Applied Torques and Mechanisms of Transience

A torsional system starting from rest will move towards a steady-state operating condition only through the application of external torques. In a typical torsional model, the external torque is applied to the motor and load inertias. Each machine has a characteristic speed-dependent torque curve which is based on its physical properties, and governs its behaviour during transience.

3.1 Electric Motor Torque-Speed Curve

A 3-phase AC electric motor has a characteristic torque-speed curve, which is based on its construction. A typical curve is shown in Figure 2.

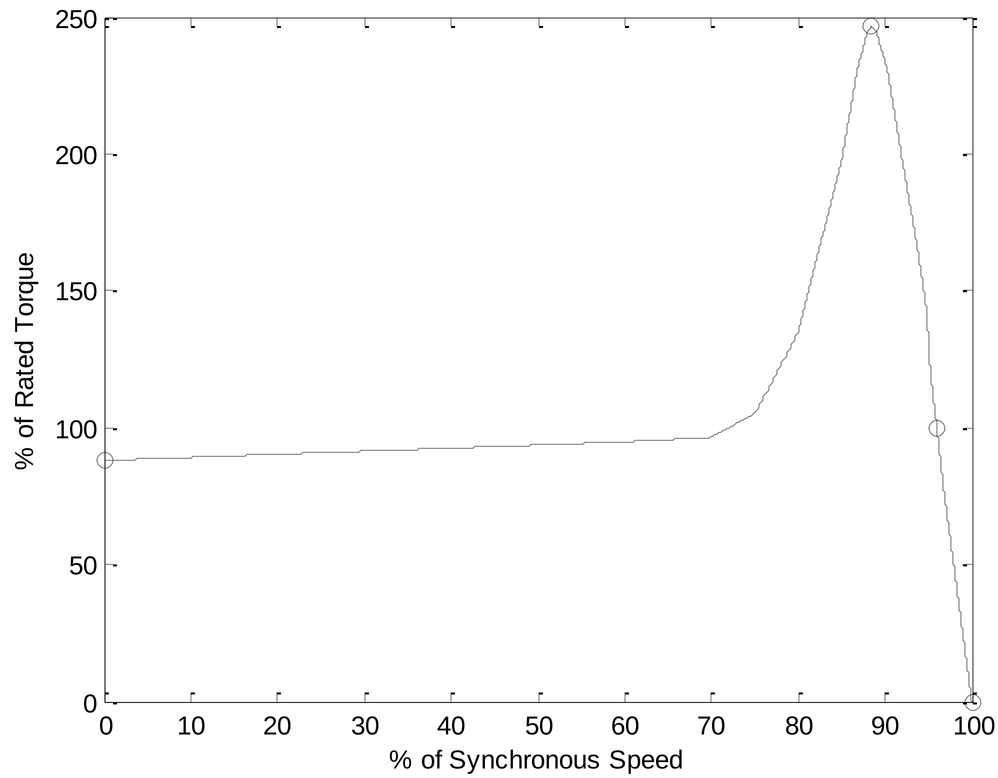

Figure 2: Typical Electric Motor Speed-Torque Curve

The shape of the curve can be obtained using field tests and measurements, or it can be calculated if sufficient information is known about the motor's electromagnetic properties. This curve gives the mean torque produced by the motor at any run speed from zero to the synchronous speed. Any higher-frequency alternating torques that may be produced by the motor are generally ignored for the purposes of the transient analysis

There are some important points in the motor torque-speed curve. The torque output by the motor at zero speed is known as the "start-up torque" and it is the torque that the motor can generate from rest. This value is important because if the torque required by the load at rest is greater than the motor's start-up torque, the system will not be able to start. The largest torque value in the curve, which generally occurs at around 90% of the synchronous speed, is called the "breakdown torque". This value is significant because during transience, the system will undergo the sharpest acceleration here, due to the large supplied torque. Finally, the point where the motor and load torque-speed curves meet is known as the "operating point", where the motor output is referred to as the "load torque". This operating point is typically only 2-5% lower the synchronous speed. Note that around the operating region of the motor curve, a large variation in torque corresponds to a very small variation in speed.

The motor's synchronous speed, denoted as N_s, which is its theoretical maximum run speed, is based on a simple equation, and is a function of the number of magnetic poles in the motor, p (typically 2, 4, 6, or 8), and the frequency of the supply line power, f in Hz. In North America and most parts of South America, this supply line frequency is 60 Hz, while in almost all other areas of the world it is 50 Hz. Eq. (10) allows for the calculation of the motor synchronous speed in units of revs/min or RPM.

$$N_s = \frac{120f}{p} \tag{10}$$

In order for a motor-driven system to operate at different steady-state speeds, the motor's synchronous speed, and its torque-speed curve, must be modified. This can be done by varying the line input frequency (since the number of poles in the motor cannot easily be modified), by use of a variable frequency drive, or VFD. This device makes use of an AC-to-AC converter to change the input power line frequency, thereby modifying the motor's torque-speed characteristics. This allows the system to run at different speeds.

The detailed effects of a VFD on operation of the system, especially during transience, will be studied in more detail in later sections.

3.2 Load Inertia Torque-Speed Curve

The exact torque-speed curve of a single-inertia torsional load is dependent on the type of machine and the individual design. However, the largest portion of the load torque is based on the torque required to keep the load rotating. The relationship between the torque and speed is quadratic, and can be specified by knowing the operating steady state speed and power (which gives the torque by dividing power by speed).

Based on the quadratic relationship, the load torque at rest would be zero. However, a starting torque is required to overcome resistances such as bearing friction, and allow the system to start.

A typical torque-speed curve for a centrifugal load such as a fan or turbine is shown in Figure 3. The relationship is quadratic, except for very low speeds, where a non-zero start-up torque is required to overcome friction.

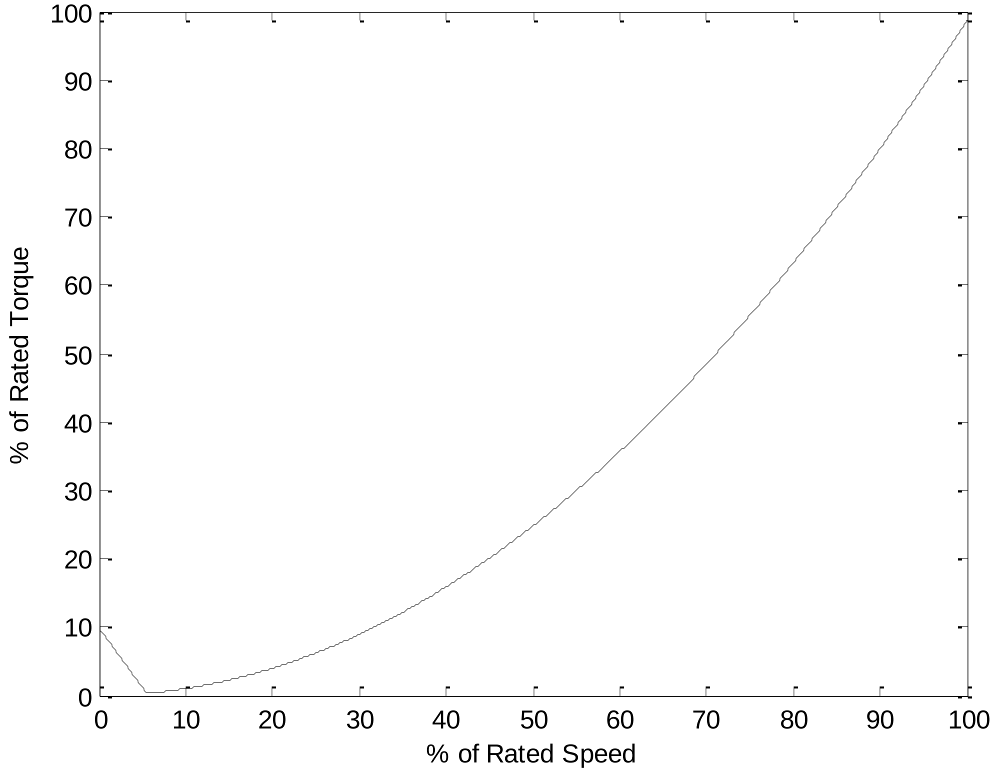

Figure 3: Typical Speed-Torque for Centrifugal Load

Note that since the load consumes power, the load torque is applied as a negative torque value in the mathematical formulation of the system.

3.3 Mechanism of Transient Behaviour

When the torsional system is started from rest, changes in speed are driven by differences in the supplied and load torques in the system.

At a certain instant in time, the speed of the system determines the motor supplied torque and load torque, according to the aforementioned torque-speed curves. In general, if the torque supplied to the system is greater than the load torque demand from the system, the drive train will accelerate, and vice versa. This self-balancing mechanism is the basis for transient behaviour, and will continue until the system reaches a speed point where the supply and demand torques are equal. This is the steady-state operating point of the system.

This description is somewhat simplified, however. Since the supply and load torques are applied to different locations in the system, other dynamic effects, particularly torsional vibrations and speed variations, will occur in the system. In this system, for example, if the motor torque output is greater than the load torque demand, the motor will tend to accelerate the whole system, but the load will naturally resist this change in speed due to its inertia. This effect will be even more pronounced for systems where the load inertia is much higher than the motor inertia.

For the system to run at different operating speeds or performance points, it is necessary to modify either the motor or load torque curves, since the system will automatically settle at the point when the two curves intersect.

The load curve can be modified by changing the performance parameters of the system, so that the machine requires more at less power at a certain run speed. However, since the slope of the motor torque-speed curve is very steep around the operating point, a large change in torque will only result in a small change in the run speed. Therefore, modifying the torque-speed curve of the load is generally ineffective when intending to run the system at different speeds.

Through use of a variable frequency drive (VFD), however, it is possible to modify the motor torque-speed curve to allow the system to run at different speeds. In addition, modification and regulation of the motor torque-speed curve by means of a VFD can be an important tool in mitigating vibration problems during start-up.

3.4 Use of Variable Frequency Drive

The motor torque-speed curve can be modified in two main ways; by variation of the input line frequency using a VFD, and regulation of the supply voltage to the motor.

The VFD, by regulating the input line frequency, directly controls the synchronous speed of the motor, and therefore can be used to modify the operating point of the system. From Eq. (10), the motor synchronous speed, N_s, is directly proportional to the line frequency.

When using the VFD to run at lower frequencies, it is common to reduce the supply voltage so that the torques are reduced as the speed range is lowered. This is done to avoid magnetic saturation of the motor core. This voltage reduction is an important factor to consider when analyzing transient behaviour.

Modifying the supply voltage has the general effect of reducing the amplitude of the torque-speed curve, in a ratio equal to the square of the voltage reduction ratio. This is due to the fact that the motor's supply torque is proportional to the square of the supply voltage. Applying 80% of the supply voltage to the motor, for example, will reduce the amplitude of the motor torques in the curve to 64% of their original values.

It will be shown in later sections that scaling of the torque-speed curve by means of supply voltage regulation can be used to reduce the severity of torsional vibrations during transience.

The effect of changes in line frequency and voltage on the motor torque-speed curve is illustrated in Figure 4. Starting from the original torque-speed curves on the far right, from right to left, the curves are shown for 90%, 75%, 50% and 25% of the original synchronous speed respectively. The torque values are reduced by a factor proportional to the squares of these speed reduction factors.

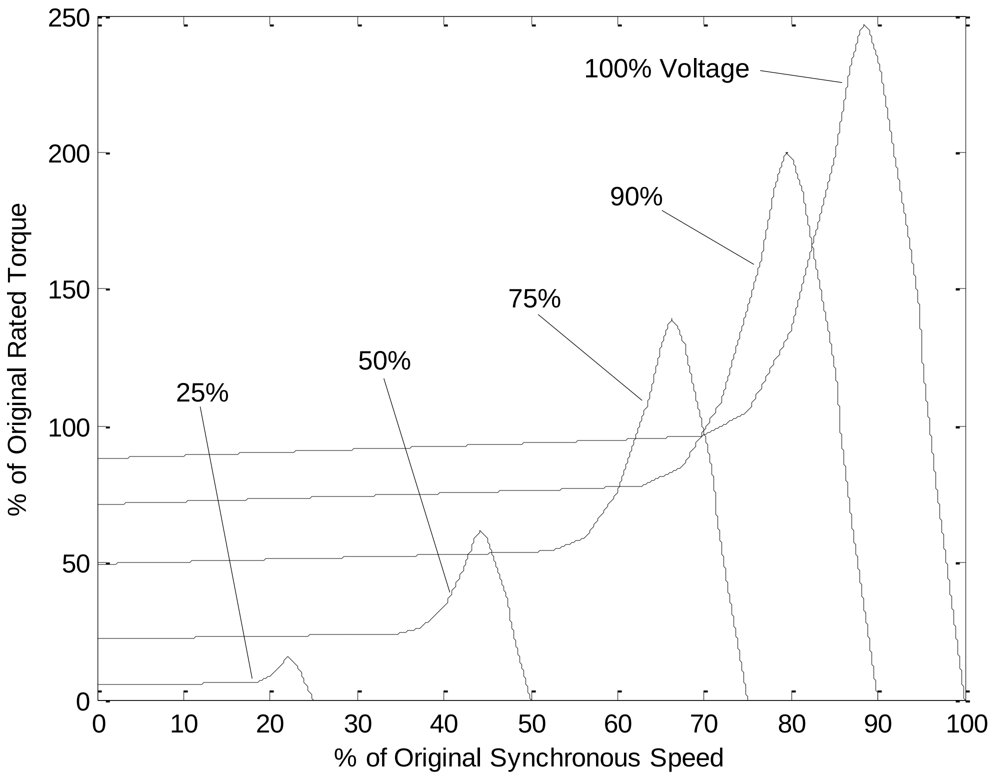

Figure 4: Illustration of VFD Effect on Motor Torque-Speed Curve

The mode of control of the line frequency during start-up is also important to consider when designing the system, as will be shown later. Failure to design the VFD start-up scheme appropriately can result in serious vibration problems.

A passive mode of start-up using the VFD is known as starting “across the line”. It involves setting the line frequency of the system to the intended final value, and allowing the system to develop through its transient state without any other external control.

4. A Transient Torsional Case Study

In this section, we introduce a case study where a transient torsional was performed on a motor driven fan unit during the design phase. The purpose of this case study, in the scope of this paper, will be to provide a base system and results from which the effect of various modifications can be studied.

The case study unit consisted of a large turbo steam fan driven by an electric induction motor. The two components were connected by a steel coupling. A transient torsional analysis was required for the system due to the very high inertia of the fan, which was almost 9 times greater than the inertia of the motor, which is an unusually high ratio, and would result in a long transient period during the unit’s start-up.

For comparison, motor-driven reciprocating compressor systems generally have a total compressor inertia which is less than one-third of the motor inertia, and in some cases even less. These systems, when started, typically reach steady-state in 2-3 seconds, so their transient behaviour is not analyzed.

4.1 Case Study Model

A torsional geometric model was prepared for the case study system as covered in section 2.1. The inertia values which were provided or calculated for the motor, fan, and coupling are shown in Table 2. The description of each inertia and stiffness value is also given; these are similar to the general descriptions provided in Table 1. The fan shaft inertia, while small compared to the fan impeller inertia, was included for accuracy, and was combined with the adjacent coupling inertia in the model. The motor and load inertias are empirically measured and provided by the manufacturer, since their complex geometry makes it difficult to calculate the rotary inertias directly, without the use of a detailed CAD model. Torque-speed curves for the motor and fan were also provided by the equipment manufacturers.

A torsional system comprised of 4 inertias connected by 3 stiffnesses will have 3 torsional natural frequencies. These were calculated to occur at 21.5, 216.0, and 597.4 Hz. These frequencies correspond to critical speeds of 1,288, 12,961, and 35,844 RPM.

Table 2: Case Study Torsional Model Parameters

Inertia Descriptor	Inertia (kg-m^2)	Torsional Stiffness (MN-m/rad)	Spring Descriptor
Motor inertia	108.9		
		4.960	Motor shaft stiffness
½ Coupling inertia	1.288		
		10.032	Coupling stiffness
½ Coupling inertia + Fan shaft inertia	3.268		
		3.874	Fan shaft stiffness
Fan inertia	925.0		

4.2 Desired Mode of Operation

The motor was rated at 5500 HP, with a rated input voltage of 6600 V, and was equipped with a VFD. The VFD operated in Volts/Hertz mode from 0 to 60 Hz, meaning that it would supply 6600 V to the motor at a line frequency of 60 Hz, but would linearly reduce the supply voltage as the line frequency was decreased. For example, at a line frequency of 30 Hz, the supply voltage would be 3300 V, and the motor torque would be scaled back by the square of this voltage turn-down ratio.

The motor has four poles, meaning the synchronous speed at a line frequency of 60 Hz is 1800 RPM. However, the fan was required to run up to 2160 RPM. For the system to be able to run at the desired maximum speed, the line frequency would have to be increased to 72 Hz.

The desired steady-state operating point of system would be at around 2150 RPM, at a full load torque of 18,590 Nm. This corresponded to a shaft power of 4186 kW, or around 5600 HP.

The intended mode of start-up for the system was to either start the system "across the line" with the line frequency initially set to 72 Hz, and allow it to reach steady-state, or to start with the line frequency at to 60 Hz, allow it to reach an intermediate equilibrium, and then increase the frequency to 72 Hz.

These two modes of operation were tested using the torsional model constructed using the parameters above. The results are reported in the following section, and serve as a base case for transient behaviour where the start-up operation of a system involves no active regulation of the system's transient behaviour.

In addition, a "coast-down" case was simulated for the system as well, to predict the transient behaviour when the power to the motor was turned off and the system was allowed to decelerate to rest. In this simulation, after the system reached steady state, the motor supply torque was set to zero at all speeds, but the load torque remained the same, since the fan would still require torque to keep spinning. The system naturally decelerated under these conditions.

4.3 Preliminary Results

Running the simulation of the mathematical model allows us to predict the behaviour of the torsional system as it develops from rest to a steady-state run condition. In the simulation, the motor curve is controlled by the line frequency and input voltage parameters, which are specified as needed based on the operating mode of the system.

The angular deflection and speed at each of the inertias was predicted as a function of time, which allowed for calculation of the twisting torques acting on each of the torsional springs, as in Eq. (4), and in turn the nominal shear stresses on the shaft components, from Eq. (5).

The two different operating modes mentioned in section 4.2, as well as a coast-down case, were simulated as baseline cases for torsional transience.

4.3.1 Start-up Case at 72 Hz

The simplest mode of start-up for the system was to set the line frequency to 72 Hz using the VFD, and allowing the system to develop to steady-state. This steady-state operating point is determined by the intersection of the motor and fan torque-speed curves. The results of running the torsional transient simulation showed very high vibrations in the system during start-up.

The angular deflection and speed for each inertia were calculated as functions of time. The torques and stresses in the system were also computed based on the system properties and angular deflections.

The instantaneous angular speed of the motor and fan over time are plotted in Figure 5. The angular speed vibrations in the motor are higher than in the fan, since the motor is lighter and requires less force to move. The system takes just over 30 seconds to reach steady state when started with the line frequency set to 72 Hz. The detail inset shows a close-up view of the speed vibrations in the motor shaft during start-up. More analysis of these vibrations will be performed in section 4.4.

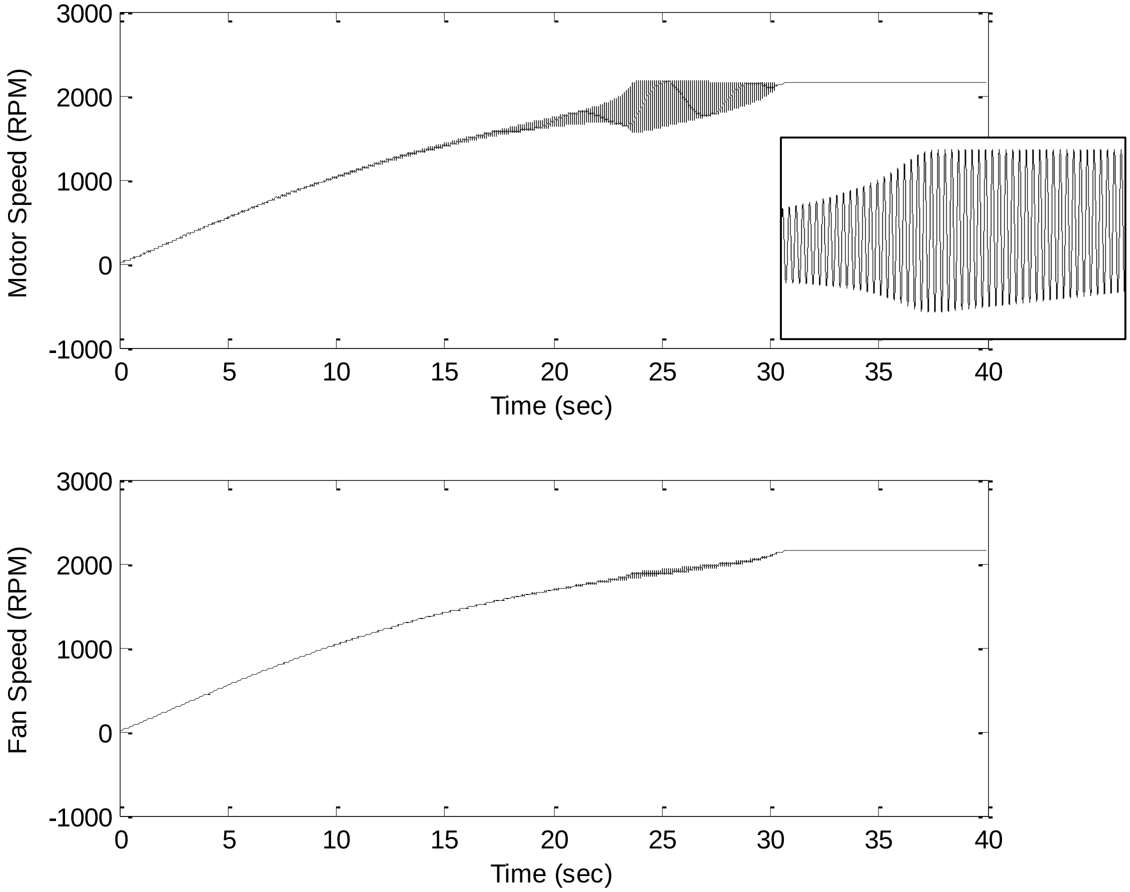

Figure 5: Speed Variation of Motor and Fan during 72 Hz Start-up

The calculated twisting torques on the motor and fan shafts are shown in Figure 6. The torques on each shaft are essentially equal, since, between the motor and fan disks, no torque is externally removed from the system, so only one plot is shown. The magnitude of the vibratory torque in each shaft is around 500,000 Nm, which is over 25 times greater than the mean operating torque. The torque waveform exhibits similar vibrations as the speed waveforms in Figure 5 do.

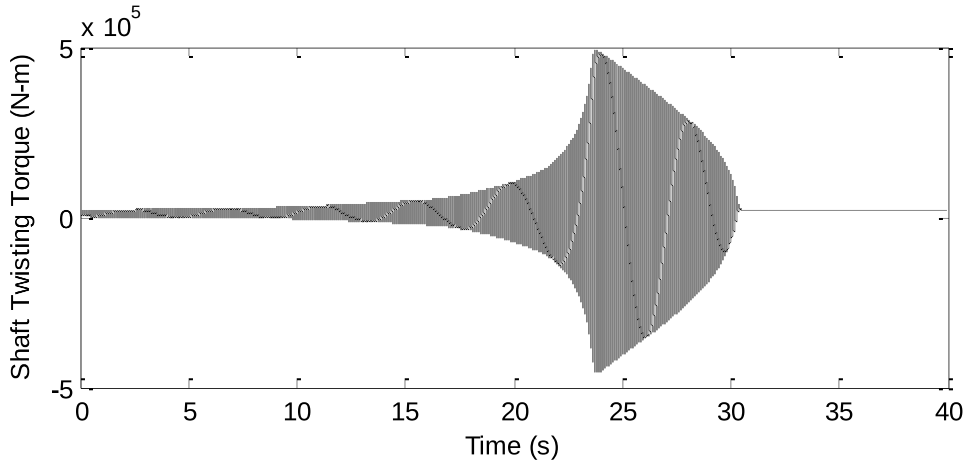

Figure 6: Twisting Torque in Motor/Fan Shaft during 72 Hz Start-up

The shaft material tensile strengths were given as approximately 448 MPa yield and 620 MPa ultimate strength for the fan shaft, and 586 MPa yield and 725 MPa ultimate strength for the motor shaft.

The von Mises stress criterion can be used to compare the stresses from a pure shear loading condition to the allowable material stresses. According to this criterion, the shear yield stress is lower than the tensile yield stress by a factor of $\sqrt{3}$. Conversely, the calculated shear stresses could be multiplied by this factor and compared to the original tensile strengths, which is what will be done in the stress calculations that follow.

The endurance limit of a material, which is the maximum alternating stress a material can tolerate, is generally calculated by multiplying its ultimate strength by a number of strength-reducing factors, such as a surface finish factor, size factor, and temperature factor. The endurance limit of a material is usually 3-5 times less than its ultimate strength.

The smallest-diameter portions of the motor and fan shafts were 180mm and 159mm (6.25") respectively. Based on the calculated vibratory torques, the nominal alternating stresses in the shafts, based on Eq. (5), would be 436.6 MPa in the fan shaft and 633.5 MPa in the motor shaft. The von Mises stresses based on these shear loads would be 756.2 MPa in the fan shaft and 1097.3 MPa in the motor shaft. These values would be even higher if the effect of stress concentrators such as diameter changes or keyways were considered [6].

Even comparing these nominal shear stresses against the ultimate shear strengths of the shafts, rather than endurance limit, which would be considerably lower still, the alternating stresses calculated during this start-up would be enough to break the shafts during start-up.

The twisting torque acting across the coupling would also be around 500,000 Nm. Again, this would be enough to easily break the coupling during an initial start-up, as the continuous torque rating for the case study coupling model was 89,000 Nm.

Clearly, starting the system across the line at 72 Hz would most likely cause catastrophic failure in the system in at least one of the components, and, having performed this initial transient analysis, it would not be recommended that the system be started in this manner at any time.

4.3.2 Start-up Case from 60 Hz to 72 Hz

An alternative proposed method of starting the system was to set the line frequency to 60 Hz, allow the system to reach an intermediate equilibrium, and then switch the line frequency to 72 Hz in order to reach the final steady-state operating point. The system takes just under 20 seconds to reach a 60 Hz equilibrium, and a further 7 seconds once the switch to 72 Hz occurs, which gives a shorter total start time than when starting at 72 Hz directly.

The speed vibrations in the motor and fan are shown in Figure 7, with the shaft twisting torques shown in Figure 8. The vibrations level are lower than for the case of starting across the line at 72 Hz, but there are now two periods of high vibrations in the system. As with the previous starting case, the speed vibrations in the fan are lower than in the motor, and the twisting torques in both shafts are more or less identical.

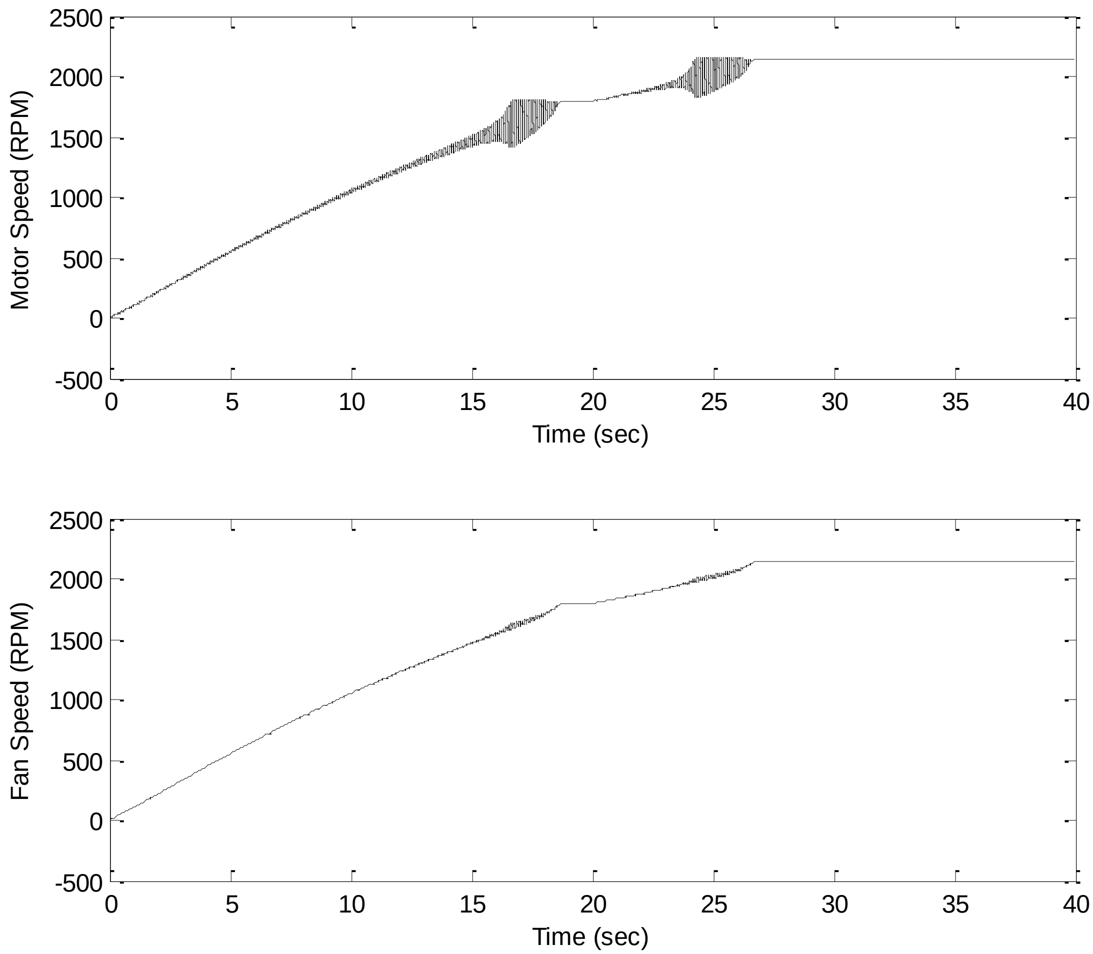

Figure 7: Speed Variation of Motor and Fan during 60 to 72 Hz Start-up

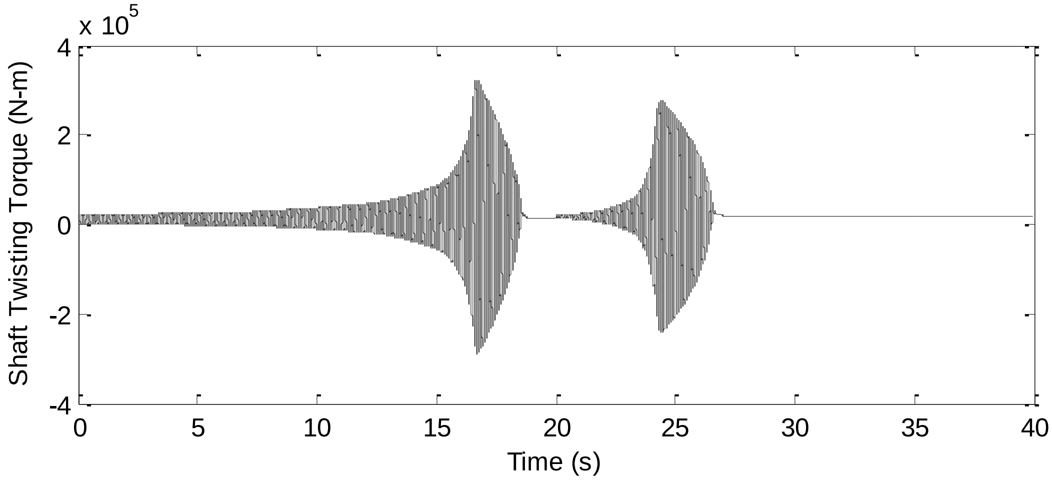

Figure 8: Twisting Torque in Motor/Fan Shaft during 60 to 72 Hz Start-up

The vibratory torques in the motor and fan shafts are around 300,000 Nm before the intermediate state, and around 250,000 Nm before the final state. These vibratory torques, while being around half of the values which were seen when starting across the line immediately at 72 Hz, are still sufficient for the stresses to yield, if not break, the shafts during start-up.

Therefore, this mode of start-up cannot be recommended for the system either.

4.3.3 Coast-down Case

The means by which the system would be stopped after operating at steady state was part of the system's transient response, and was therefore also analyzed. The system was intended to be stopped by cutting the power to the motor and allowing it to coast down to rest.

With the motor torque set to zero at all speeds, the torque demand by the fan at the operating speed would not be met, and the system would naturally decelerate. Once the system reached 5% of its steady-state speed, a nominal braking torque was applied to the system to simulate the additional effect of friction, and to avoid an asymptotic solution.

To simulate the coast-down, the system was first started across the line up to steady state. The motor torque was then set to zero, and the system was allowed to decelerate. The speed vibrations in the motor and fan are shown in Figure 9. The system took approximately 240 seconds to reach 5% of its steady-state operating speed.

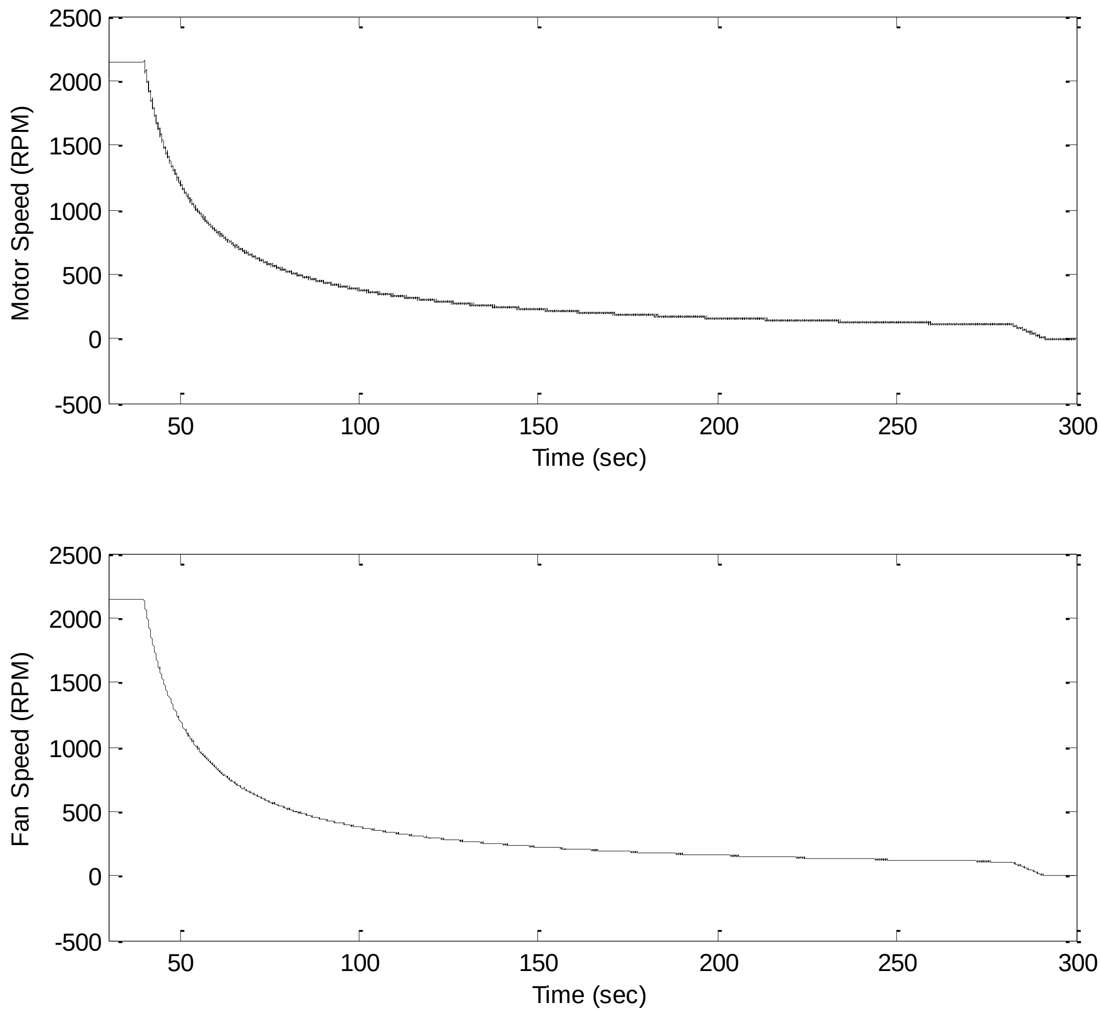

Figure 9: Speed Variation of Motor and Fan during Coast-down

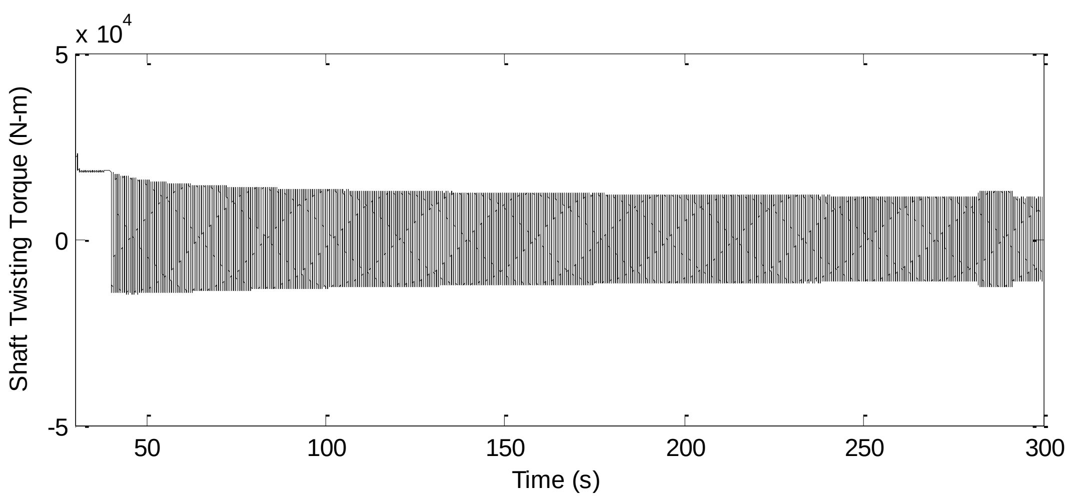

Figure 10: Twisting Torque in Motor/Fan Shaft during Coast-down

The vibratory torques during coast-down are much lower than during start-up, and are under 20,000 Nm. These loads are on the same order of magnitude as the mean operating torque, and would result in equivalent von Mises stresses of approximately 44 MPa in the fan shaft and 30 MPa in the motor shaft. These stresses are well within the endurance limit of the shafts, as they are less than 10% of the shaft material shear strengths. Therefore, cutting the power to the motor and allowing it to coast down is an acceptable means of stopping the system.

4.4 Assessment of System Performance

In the two start-up cases analyzed, the predicted torsional vibrations were very high during some portions of the transient response. The torques and stresses in the system in either base case were well above the allowable limits for the motor and fan shafts, and for the coupling.

In the speed vs. time plots in Figure 5 and Figure 7, the high vibrations are isolated to the time period just before the system reaches steady state. This time corresponds to the system going the section of the motor torque-speed curve around where the breakdown torque, or maximum torque, occurs. At speeds around the breakdown point, the torque varies sharply with small variations in speed, which is one of the driving factors in the vibrations in the system, as variations in speed cause large changes in the motor torque, which in turn causes further speed variations.

This instability effect is exacerbated by the fact that the fan is much heavier than the motor, and is therefore much more resistant to the motor's tendency to accelerate the system than a lighter mass would be.

If the system is run with the exact same physical properties and applied torques, with the fan inertia reduced to 100 kg-m^2, which is around the same as the motor inertia, the maximum vibratory torque in the system with 72 Hz start-up is reduced to less than 10,000 N-m. The speed vibrations which were present in the original system become effectively non-existent. The same result holds true if the motor inertia is increased to 900 kg-m^2, which is almost equal to the fan inertia. This suggests that these high transient vibrations and torques are a direct result of the large ratio of the fan inertia to the motor inertia.

Another factor which contributes to the high transient vibrations is the relatively high stiffness of the system components, particularly the coupling. Since the coupling stiffness is of the same order of magnitude, and is in fact higher, than the adjacent shaft stiffnesses, vibrations are readily transferred between the motor and fan, and have no opportunity to be damped out either.

Performing a simple Fast Fourier Transform (FFT) on the transient vibration waveform allows us to see the frequency content of the vibrations. Since the only external torques are mean torques from the motor and fan, no forcing harmonic components are being applied to the system. Therefore, the system, as it would in a free modal response, should vibrate at its torsional natural frequencies, with the acceleration torques applied during start-up acting as a sort of broadband excitation. Indeed, the FFT of the transient waveform shows a sharp frequency spike exactly at the first natural frequency of the case study system, at 21.5 Hz, shown in Figure 11.

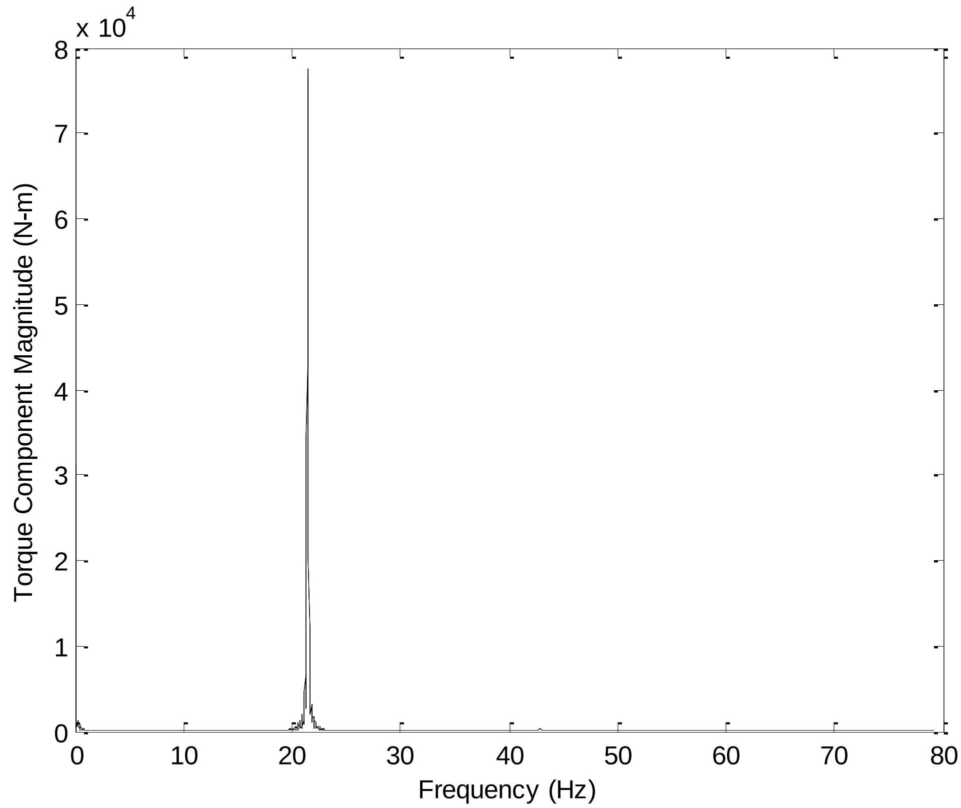

Figure 11: Frequency Content of Transient Torsional Vibrations

When assessing the performance of the torsional drive train components, the torques or stresses can be used to determine whether certain components are acceptable for the system under the given loads and operating conditions. For custom-made parts with complex geometry, such as the coupling, the manufacturer will usually give torque limits for the part. However, for assessing drive shafts, it is more appropriate to use the torque loads, geometry, and material strengths, to calculate the stresses, which can then be used as an evaluation criterion. To calculate the nominal stress levels, Eq. (5) may be used; however, this does not take into account the effects of geometric stress concentration factors.

For analyzing the effect of alternating stresses on a component, fatigue effects must be taken into account. There are many criterion for evaluating the acceptability of a state of stress on a system. Two prominently used criteria include the Modified Goodman diagram and the S-N curve.

The Modified Goodman diagram is used for evaluating a combined state of mean and alternating stresses, compared to infinite fatigue life limits. The infinite life criterion is based on the material's ultimate strength and fatigue strength. The fatigue strength is calculated by multiplying the ultimate strength by a number of reducing factors which are based on geometry, manufacturing methods, and operating conditions.

While useful for steady-state analyses, the Goodman diagram is used as a "yes/no" criterion as to whether the shaft meets the infinite life criterion. For transient analyses, where the loads are only expected to be applied for a finite amount of cycles, the S-N curve is generally a more appropriate assessment tool. This curve plots allowable number of load cycles (N) against the alternating stress (S) to estimate fatigue life.

For ductile materials such as various types of steels, a simple S-N curve can be constructed to estimate the number of allowable load cycles that a certain component can withstand, under a given alternating stress. A simple example of an S-N curve is shown in Figure 12. The maximum allowable stress at 10^3 cycles is usually defined as 0.8 times the material ultimate tensile strength, and the allowable stress at 10^6 cycles is equal to the endurance limit stress. In the plot axis, S_u is the ultimate tensile strength, S_e is the endurance limit, and S_{alt} is the calculated alternating stress on the component. If the alternating stress is lower than the endurance limit stress, the component will theoretically be able to operate for an infinite number of cycles. The example alternating stress which is plotted in Figure 12 would correspond to an allowable number of cycles of just under 1.0E+5.

The S-N curve does not take into account the probabilistic factors involved in fatigue life assessment, such as the presence of existing cracks, material grain structure, and operating temperature.

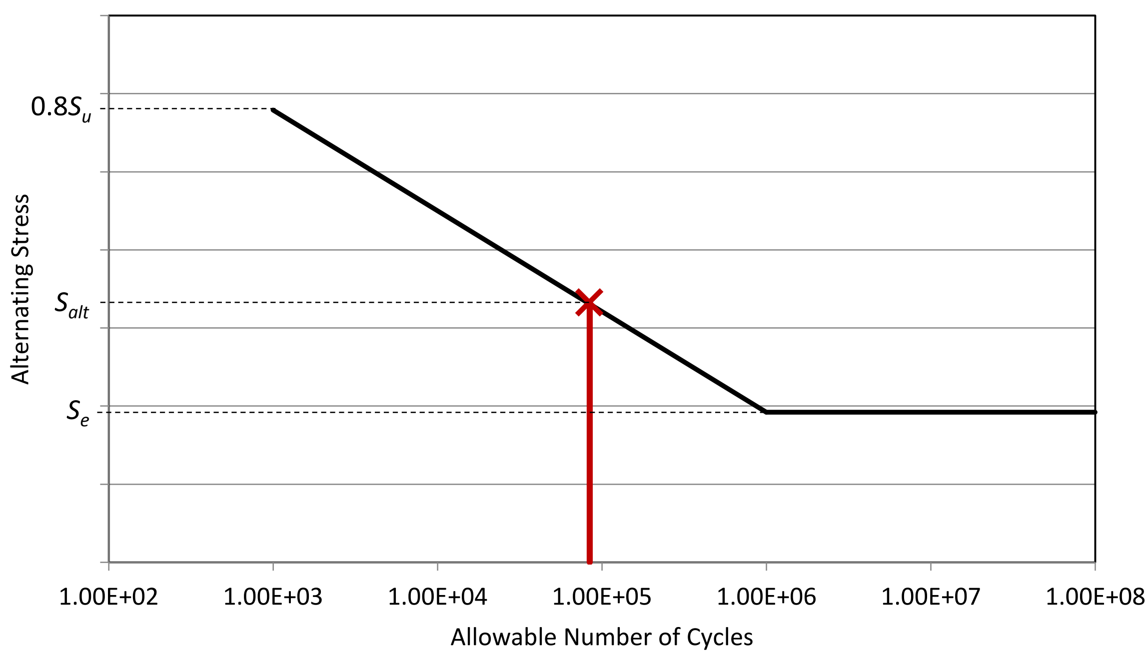

Figure 12: Example S-N Curve

The S-N curve can be used as a design tool, by which the maximum allowable loads and stresses on the component can be calculated, given a desired number of life cycles which the component should be able to withstand. For example, if a machine is required to start once a week for 20 years, this will be around 1000 start-ups required over its lifetime. For each start-up, it is important to predict the number of actual fatigue cycles the machine will experience to assess the cumulative damage. For example, the start-up across the line shown in Figure 6 corresponds to around 200 cycles. This will mean that the machine will have to withstand around 2E+5 fatigue cycles over its intended lifespan.

Another complication in the calculation is that, as in the twisting torque plot in Figure 6, not all cycles during the start-up are of the same magnitude. In this case, assigning all load cycles to have the maximum magnitude will results in a conservative result when calculating the allowable loads and stresses. In this case, a piece-wise computational method such as Miner's rule [7] may be used to account for the cumulative effect of load cycles of different magnitudes.

For practical purposes, it is important to investigate general methods by which these high torsional vibrations can be mitigated through feasible solutions. The solutions and modifications which will be investigated include physical changes to the system equipment, as well as changes to the operating mode in which the unit is started.

5. Mitigation of Torsional Vibrations during Start-up

The results of the start-up runs presented in the case study are clearly unacceptable for any real system which is to be constructed and operated. In this section, using the case study runs as a starting point, we will present some general modifications to both the system and start-up operating mode with the intent of reducing the vibrations during start-up, and, by extension, the system transient torques and stresses, to acceptable levels.

5.1 Driver-Load Inertia Matching

As mentioned in section 4.4, the high torsional vibrations in the system do not occur if the motor and fan have roughly the same inertia. However, implementing this modification is likely to be difficult, and may cause further problems.

Upsizing the motor to one with a larger inertia would create problems in terms of matching the power of the motor and load. A much larger motor would also have a higher torque output, and therefore would accelerate the system more quickly. The test runs with the matched motor and load inertias were done without any modifications being made to either torque-speed curve. In addition, it may not be practical to buy a completely new piece of machinery for the system.

The other method of theoretically implementing this modification would be to add inertia disks to the motor's rotor. However, due to the actual construction of the rotor, which includes large magnetic bars and windings, this is not a practical solution.

The load is usually selected for its performance parameters within the scope of a larger system or process, such as flow rate, work production, or energy output. Downsizing the load machine to reduce its inertia would disrupt the operation of whatever larger scope system the load is a component of.

Therefore, this method of matching the driver and load inertias to reduce start-up vibrations is likely to be unfeasible when designing a real system.

5.2 Gradual Line Frequency Increase

The case study runs in section 4.3 were based on starting the system “across the line” at either 60 or 72 Hz, and the predicted torques and stresses would be sufficient to break the shafts and coupling during the system’s initial start-up.

The highest vibrations during start-up are due to the system passing through the motor’s breakdown torque, which is the area of highest torques in the motor torque-speed curve. When the system is started across the line, this passing is done quickly, without regulation, and occurs with the motor torques at their full value.

The VFD is often used in practical applications to start up the system slowly by gradually increasing the line frequency. While the system will take longer to start up, the torsional vibrations, torques, and stresses during transience can be reduced if the mode of start-up is effectively designed.

In addition, the VFD in the simulation was considered to run in “Volts/Hz” mode up to 60 Hz, which was mentioned briefly in section 3.4. Therefore, at lower frequencies, the applied voltage to the motor will be lower, as will the supply torque.

Figure 13 shows the speed variation over time of the motor and fan when starting the system at 10 Hz, and increasing the line frequency by 0.5 Hz/s. Note that, although the system takes longer to start than when starting across the line, the speed vibrations during transience are much lower. This is due to the fact that the system is able to pass through the unstable region of the motor curve at lower speeds and torques, as the line frequency is slowly ramped up. Once this region has been passed, the speed and torque increase linearly and stably, as the system remains within the linear region of the motor curve, even as the curve is being modified by the VFD.

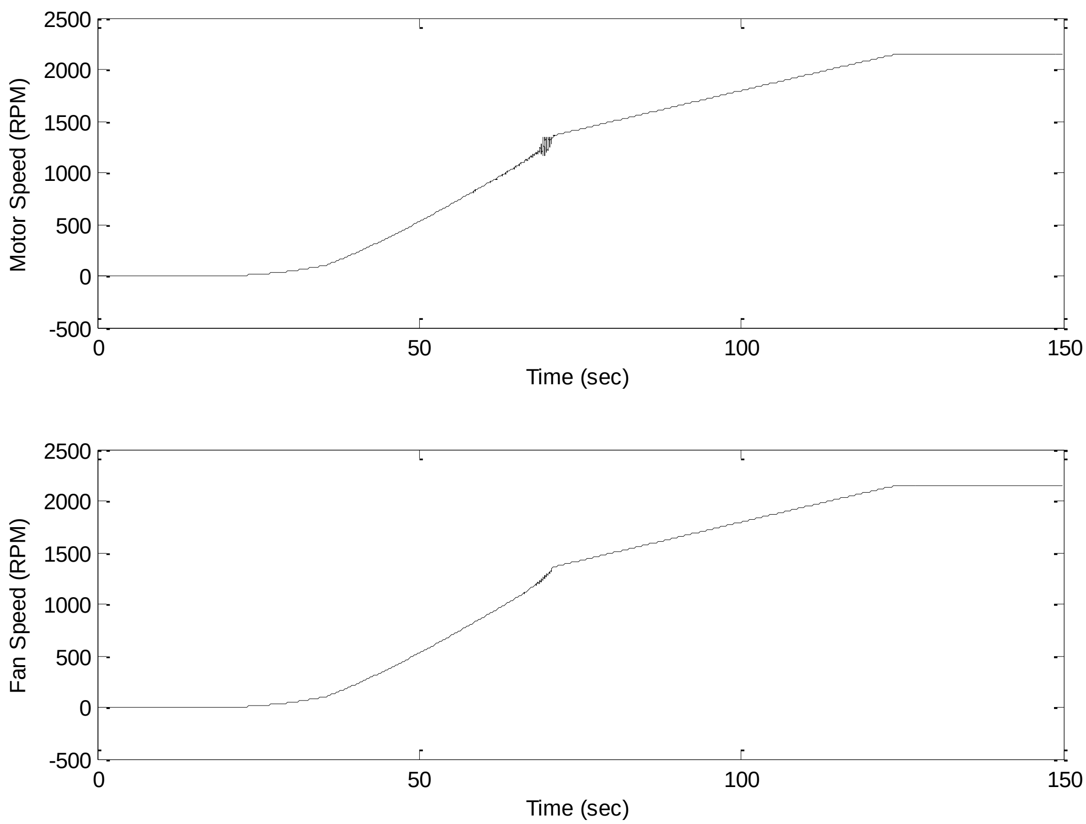

Figure 13: Speed Variation of Motor and Fan during Start-up from 10 to 72 Hz, 0.5 Hz/s Increase

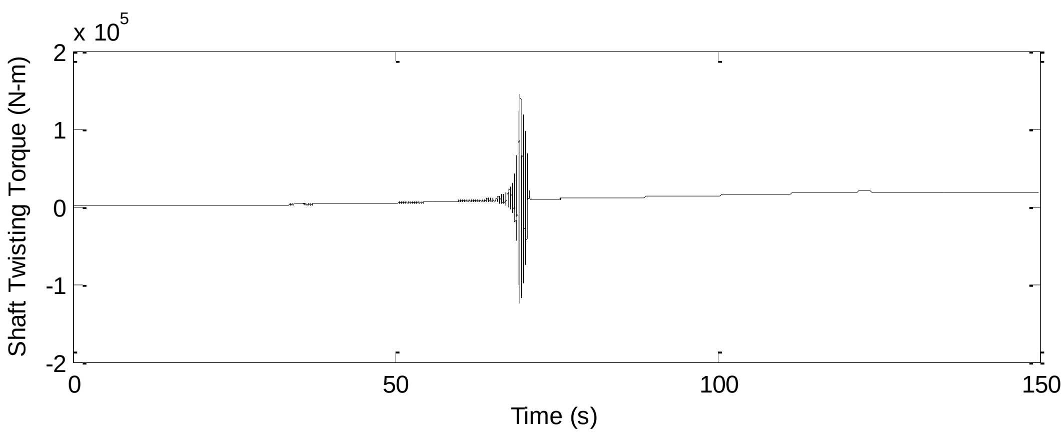

Figure 14: Twisting Torque in Shafts during Start-up from 10 to 72 Hz, 0.5 Hz/s Increase

Table 3 shows the results of several tests runs where different starting frequencies and line frequency increase rates (in Hz/s) were used. The time to reach steady-state and the maximum vibratory torque during transience are reported for each test case. The start-up times are rounded to the nearest second, and vibratory torque values are rounded to the nearest kN-m. The vibratory torque is defined as the peak maximum torque on one cycle minus the successive peak minimum, divided by two. For comparison, recall that the mean torque at the full load operating point is 18.6 kN-m.

Table 3: Maximum Vibratory Torques (kN-m) for Gradual Start-up Cases

Start Frequency (Hz)	Frequency Increase Rate (Hz/s)	Start-up Time (s)	Max Vibratory Torque (kN-m)
5	0.1	670	68
5	0.25	270	95
5	0.5	135	126
5	1	68	178
5	2	49	241
10	0.1	621	67
10	0.25	251	94
10	0.5	124	132
10	1	63	177
10	2	47	271
15	0.1	574	68
15	0.25	230	93
15	0.5	115	125
15	1	58	156

These results show that a gradual increase of the line frequency is an effective method in reducing the vibratory torques during start-up. By starting the system at a lower line frequency, it is also possible to have the system pass through the motor curve’s breakdown region when the torque values are lower, due to the VFD’s Volts/Hz mode at lower frequencies.

The main factor in the maximum vibratory torque during start-up is the frequency increase rate, rather than the start frequency.

A certain start-up mode can be programmed into the VFD so that the line frequency is regulated during the system’s transience. This solution is relatively simple to implement, and does not require purchase of new equipment or physical modifications to the system, as most VFD come equipped with this programmable capability.

However, one drawback to using this method to control torsional vibrations during start-up is the inherent unpredictability and unreliability involved in software-based solutions. In a system like the one presented in the case study, if the programming of the VFD start-up were to fail or be improperly implemented even once, the system components would most likely fail due to the very high start-up vibrations when starting across the line.

Therefore, for a general design case, it is important to investigate further options which will add to the robustness of the system design, and will allow the system to operate more safely and reliably during start-up.

5.3 Specialized Coupling Selection

The case study system had a steel coupling selected for the initial system. The selected type and model of coupling is often referred to as "rigid" if its stiffness is of the same order of magnitude as the surrounding shaft stiffnesses. Indeed, for this particular coupling, the torsional system parameters in Table 1 show that its stiffness is more than twice the stiffness of the motor and fan shafts.

Since the coupling has a stiffness comparable to the surrounding shafts, the coupling will readily transmit any vibrations and motion between the motor and fan. This becomes problematic due to the very large inertia of the fan, since it will resist the driving motion of the motor by means of reactive torques, which will be directly "felt" by the smaller motor. In a system where the load is small, a rigid coupling has no negative effect, since the motor can easily accelerate the lighter inertia load as it needs to. However, with a very high inertia load as in the case study unit, the rigid coupling causes the motor to directly try to push against the fan, which, due to its inertia, exerts high reaction torques against the motor.

It is common, for systems where torsional vibrations are a problem, to make use of a specialized coupling. These types of coupling are designed to have a very low stiffness, and also sometimes have an inherent damping component as well due to their construction.

These specialized coupling often make use of rubber components, which are designed to be compressed under torsional load. These elements have low stiffness and also provide some damping.

To test the effectiveness of these types of coupling in reducing start-up torsional vibrations in a general system, transient runs have been performed with a variety of hypothetical combinations of stiffness and damping for the coupling.

The results in Table 4 and Table 5 show the effects of a coupling with low stiffness and added damping on the maximum vibratory torque during start-up. The coupling inertia was not modified for any of these run cases. The vibratory torques are reported for start-up across the line at 72 Hz and for start-up with an initial frequency of 10 Hz and a line frequency increase of 0.5 Hz/s. The change in coupling parameters does not significantly affect the start-up time.

Table 4: Maximum Vibratory Torques (kN-m) vs Coupling Parameters, 72 Hz Start-up

Coupling Damping (N-m/(rad/s))	**Coupling Stiffness (MN-m/rad)**				
	4.0	**2.0**	**1.0**	**0.5**	**0.25**
0	432	384	326	267	213
25	402	327	248	182	132
50	375	286	194	124	82
100	333	210	114	45	12

Table 5: Maximum Vibratory Torques (kN-m) vs Coupling Parameters, Slow Start-up

Coupling Damping (N-m/(rad/s))	**Coupling Stiffness (MN-m/rad)**				
	4.0	**2.0**	**1.0**	**0.5**	**0.25**
0	111	97	88	69	58
25	102	79	56	34	22
50	91	56	25	3	6
100	67	17	3	2	3

Figure 15 and Figure 16 show the speed and twisting torques in the system during a slow start-up where a softer coupling with damping is used. Note that the high speed and torque vibrations which were seen in prior case runs are essentially non-existent here, and the system is still able to reach its maximum operating speed in around two minutes.

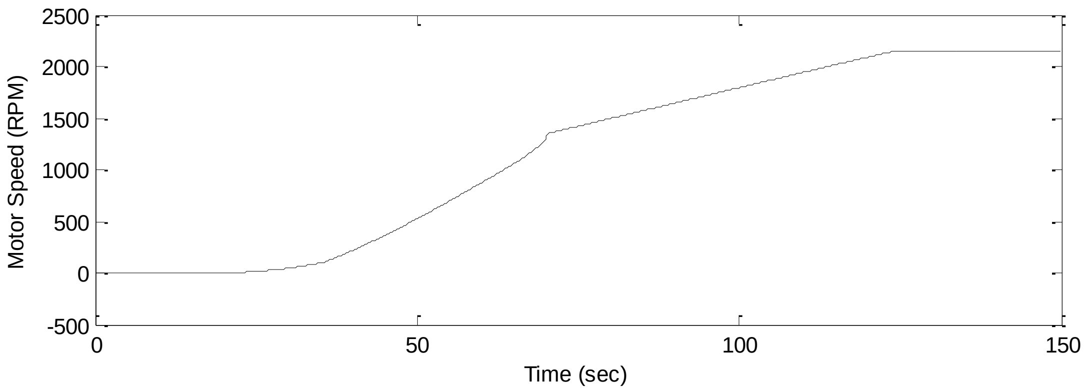

Figure 15: Speed Variation of Motor during Slow Start-up with Coupling Stiffness of 0.5 MN-m/rad and Damping of 50 N-m/(rad/s)

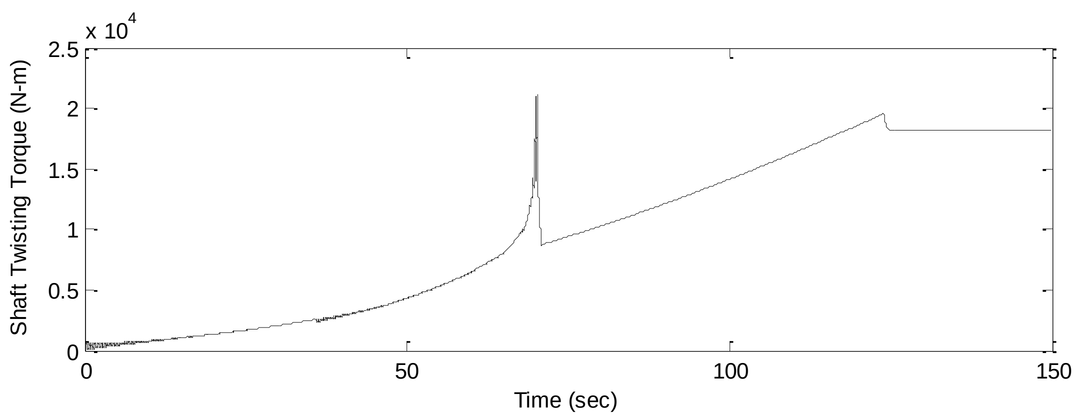

Figure 16: Twisting Torque of Shafts during Slow Start-up with Coupling Stiffness of 0.5 MN-m/rad and Damping of 50 N-m/(rad/s)

These runs show that a soft coupling with damping is highly effective in reducing the torsional vibrations in the system during transience, even in the case where the system is started across the line. Given an acceptable range of stiffness and damping parameters based on similar transient torsional runs, an appropriate coupling selected could be made in the design of a real system. It is important to note that a coupling with low stiffness, combined with a damping component, provides the most significant reduction of transient vibrations in the system.

The results in Table 4 and Table 5, as well as in Figure 15 and Figure 16 show that combining a specialized coupling in the system with a well-designed start-up mode is highly effective in reducing vibrations and torques during transience.

6. Conclusions

This paper has presented a general study of the start-up behaviour of a torsional system with a very high-inertia load. While standard torsional analyses generally focus on steady-state operating cases, systems with very high-inertia load pose unique design challenges which can only be fully studied using a transient torsional analysis.

After presenting a case study of a torsional system with problematic transient behaviour, modifications were made to the system to test their effectiveness in mitigating torsional vibrations. It was shown that starting the system slowly by using the VFD to regulate the line frequency, as well as selecting for the system a specialized coupling with a low stiffness and some damping were both successful and practical strategies in reducing the vibrations and torques seen during start-up. A combined implementation of these two solutions was demonstrated to be particularly effective.

For torsional systems where the load inertia is significantly larger than the driver inertia, a detailed transient torsional analysis can identify problems which may occur during start-up, and provide a means of designing modifications and solutions in order to mitigate these issues.

7. References

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