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Abstract:

Bolts are the most common form of connection between engineered components. However, these connections present a degree of complexity that is most often lost in the everyday practice of twisting wrenches. This study analyzes the detailed stress distribution for different bolt loading situations with either simple pulling action or preload clamping requirements. It also investigates the use of diverse material pairs for the internal and external threaded components and its impact in the consequent patterns of loading.

A closed-form analytical model for thread load calculation was created and compared with NX Nastran, a general purpose Finite Element (FE) analysis package. The model was validated against known data available in the literature. Important observations were made with respect to the load distribution of the engaged threads, and to the effect of varying the engaged length or stiffness ratio between external and internal threaded components.

Some of the more well known techniques utilized in shifting the load distribution of the engaged threads are also discussed.

Keywords: Load Distribution, Threaded Connection, Bolted Connection, MSC Nastran

1. Introduction

It has been widely accepted that in a threaded connection the first few engaged threads hold most of the load (Grewel, 2009). However, the detailed distribution of force and stress along the entire thread is relevant to many problems and the availability of a tool that can be used for determining the stress distribution will certainly be helpful in the design of threaded connections as well as in the analysis of problems that may arise whenever they are used.

The purpose of this study is to develop a structural model of a threaded connection which allows for the efficient estimation of load distribution within the threads. Finite Element Analysis (FEA) can be used for the calculation of load distributions, but since detailed FEA is both time consuming and expensive, the development of a simplified closed form analytical model was sought for this purpose.

Load distribution of a threaded connection varies because of:

- 1. Geometry of internal (male) and external (female) components
 - a. Must consider effective engagement areas to calculate real stiffness
- 2. Material properties of internal and external components. Two combinations were studied:
 - a. Case 1: Both materials with the same stiffness
 - b. Case 2: Male thread material is stiffer (i.e., steel) than the female threaded material, which is made more compliant (i.e., aluminum block)
- 3. Type of load: Simple pulling vs. clamping

An analytical model was developed to estimate the percentage load supported by each thread in a connection. These results were compared to the Finite Element model built using NX Nastran. Two loading cases were considered:

- 1. A simple 'pulling' action is applied to the external threaded component.
- 2. A clamping, pre-load type situation

2. Methods

Description of the two models:

- 1. Simplified Structural Model
 - a. The threaded connection is broken down into discrete structural components and stiffness is calculated for each of these structural components. A recursive stiffness algorithm is used to compute the forces and displacements that each thread experiences in the axial direction. Please refer to section 3 for details.
- 2. FEA 2D (Axisymmetric) Model

In this FEA study, a threaded connection is represented by a wedge shaped geometry meshed with solid elements; linear gap elements are used to represent contact connection between the internal and external threads and loads are applied as pressure at the appropriate surfaces. Constraint boundary conditions are applied to fix the bottom of the

'nut' or 'block' part from movement; appropriate constraints are applied at side surfaces to account for the wedge shape in the model. The program outputs the forces normal to the surface of each tooth. The axial component of the output forces are calculated using geometry and are used for comparison with the simplified structural model. For comparison purposes, displacements and forces reported from the FEA model are used to calculate stiffness of various components that are used in the Simplified Structural Model. Please refer to sections 3 and 4 for details of where displacement results are reported from the FEA model, and how the stiffness is calculated for each of these structural components.

For simplicity, the internal threaded (female) component will be referred to as a 'nut' and the external threaded (male) component will be referred to as a 'bolt'. Figure 1 shows how this helical geometry was simplified as an axis-symmetric 2-D shape.



Figure 1: (Helical) Geometry Simplifications

In the Simplified Structural Model, the threaded connection is represented by a network of springs, as shown in Figure 2. Springs on the left side represent stiffness of the nut. Springs on the right hand side represent stiffness of the bolt. Effective stiffness of the connection between the bolt and nut springs are represented by springs shown in the center. The center springs are connected to the 'bolt' and 'nut' springs via rigid links. Dots represent spring attachment points. Figure 3 shows the location of the points in the FEA Model that were used for calculation of deflections and stiffness. The following sections show how these measurements are used for calculation purposes, and how the model is partitioned into discrete components.



Figure 2: Structural Components of Threaded Connection



Figure 3: Reporting Locations from FEA

Figure 4 shows a Free Body Diagram (FBD) for 'Pulling Type Loading' conditions. Figure 5 shows a FBD for 'Clamped Type Loading' conditions. Note that in both figures only the axial forces were drawn. In reality, since the normal vectors of the contact surfaces of the threads do not point in the axial direction, there would be negative radial forces on the bolt and positive radial forces on the nut. However, since we are dealing with an axis-symmetric geometry, as long as we note that the bolt would undergo some compression, and the nut would be pushed outwards, it isn't necessary to focus on these radial forces for our calculations.



3. Detailed Model:

The following formula is used to calculate both the nut and bolt shank section stiffness:

$$k = \frac{A * E}{L}$$

For the bolt shank sections, the effective area used was the cross-sectional area up to, but not including the bolt threads. Likewise, the effective area used for the nut shank sections included all of the cross-sectional area up to the root of the nut threads. If significantly larger nut geometry was used, then not all of the area near the outer radius of the nut would be used.

Although it is possible to use a hand calculation method to calculate the nut and bolt thread stiffness, it would likely be more work and less accurate than running a very simple FEA. Therefore, one of the pairs of threads from the FEA model used in this analysis was taken and analyzed. The shank sections that were attached to those threads were kept, but boundary conditions and material properties were modified such that the shanks were not allowed to rotate. That is, these shank sections were approximated as rigid bodies, with $E=1x10^8$ GPa. All degrees of freedom on the bolt shank section were constrained, and the nut shank section was only allowed to move in the axial direction. Therefore, when the force was applied to the top of the nut, displacement of the nut shank section could be used to calculate the combined stiffness of the threaded connection. To calculate bolt thread stiffness, the nut thread was modified to have rigid properties.

For comparison purposes, displacements reported from the FEA model were used to calculate stiffness for each of the discrete structural components. The following formulas in Table-1 were used for deflection and stiffness calculations. A thread is treated as a cantilever beam that is mounted to its respective shank section. Therefore, when calculating the stiffness of the thread, it is intuitive to consider the deflection of the thread at the application of the force with respect to where the thread is anchored to the shank section. However, if no bending were to occur in the FEA model, then it would be equivalent to consider the deflection. Thread forces were calculated in this manner to assess whether the simplifications made for the Simplified Model are valid for all of the loading conditions. Since the Simplified Model does not account for the nut or bolt deflecting in the radial directions, it was valuable to incorporate bending into the thread calculations to better illustrate what was happening for each case.

Sections	Loading	Stiffness	Deflection		Legend
	Clamped	$F = \sum^{j=i} F$		F _b	Force on Bolt
	Loading	$kb_i = \frac{\Gamma_b - \sum_{j=1} \Gamma_j}{1 - \sum_{j=1} \Gamma_j}$		F _n	Force on Nut
Bolt Shank		$ds_i - ds_{i+1}$	$ds_i - ds_{i+1}$	ds _i	Displacement of
	Pulling Type				Bolt or Nut
		<i>i_i</i>			Shank Section
	Clamped	$\sum_{j=1}^{j=i} F_j - F_n$		44	l Disale serve ant of
	Loading	$kn_i = ds_i - ds_{i+1}$		at _i	Displacement of
Nut Shank	8		$ds_{i} - ds_{i}$		BOIL OF NUL THREAD
Nut Shank		$\sum_{i=1}^{j=i} F_j$	$us_l us_{l+1}$	F.	ε Force at
	Pulling Type	$kn_i = \frac{1}{ds_i - ds_{i+1}}$		- 1	i
				kb _i	Bolt Shank
	Clamped			-	Stiffness at
Bolt	Loading	$kbt_i = \frac{-F_i}{-F_i}$	$dt_i - ds_i$		i
Threads		$dt_i - ds_i$		kn _i	Nut Shank
	Pulling Type				Stiffness at
	Clampad			1.6+	l Dolt Throad
	Loading	F_i		KDL _i	Stiffnoss at
Nut Threads	Luaung	$knt_i = \frac{t}{dt_i - ds_i}$	$dt_i - ds_i$		i
	Pulling Type			knt;	Nut Thread
	0 //				Stiffness at
	Clamped				i
Combined	Loading	$keq_i = \frac{kbt_i * knt_i}{kbt_i + knt_i}$	$dt_i - ds_i$	keq _i	Combined Thread
Threads					Stiffness at
	Pulling Type				i

Table-1: FEA Model: Deflection and Stiffness Formulas

4. Results/Discussion

_	Case	Bolt Material	'Nut' Material	Loading Conditions
	1P	Steel	Steel	Pulling
	1C	Steel	Steel	Clamping
	2P	Steel	Aluminum	Pulling
	2C	Steel	Aluminum	Clamping

Table -2: Cases Analyzed

Table -2 shows a list of all the cases that were analyzed.

Figure 6 shows an exaggerated view of the total displacement for Case 1P. The darker, black shading
represents the undeformed shape, while the lighter grey shade represents a deformed view that is scaled
up 5000 times. Since the nut deflects away from the bolt, and since the bolt compresses radially, the
axially deflection of the first nut thread does not equal the axial deflection of the first bolt thread. This
differential deformation is most prominent in the area containing the first few threads.



Figure 6: Case 1P: FEA Model: Total Displacement for Thread #1

Error! Not a valid bookmark self-reference. summarizes the displacement reported at thread #1 for each of the cases. For Case 1C, both the bolt and nut axial displacements are at least one order of magnitude higher than their respective radial displacements. Therefore, it can be expected that the distortion in bending is less severe in this case. Since the Simplified Structural Model doesn't account for bending, it is likely that Case 1C will most closely match FEA Results.

Table -3: FEA Model: Thread #1 Displacement Summary

Case	Bolt Axial	Nut Axial	Bolt Radial	Nut Radial

	Displacement [m]	Displacement [m]	Displacement [m]	Displacement [m]
1P	9.17x10 ⁻⁸	8.07x10 ⁻⁸	-7.98x10 ⁻⁹	1.15x10 ⁻⁸
1C	2.36×10^{-7}	2.18×10^{-7}	-7.72x10 ⁻⁹	2.30×10^{-8}
2P	$1.17 \mathrm{x} 10^{-8}$	9.96x10 ⁻⁹	-8.10x10 ⁻⁹	1.54×10^{-8}
2 C	4.32×10^{-8}	1.33x10 ⁻⁸	-7.92x10 ⁻⁹	4.20×10^{-8}

Figure 7 shows the bolt and nut shank section deflections for Case 1P that were reported from the FEA Model.



Figure 7: Case 1P: FEA Model: Shank Section Deflections

Figure 8 shows the bolt and nut shank section stiffness for Case 1P that were reported from the FEA Model. Although both parts are made of steel, the nut has a much higher (shank) stiffness because the nut has a significantly larger cross-sectional area. Calculations seem to indicate that the first few shank sections on the nut are much stiffer, relatively speaking. However, this counter-intuitive phenomenon is a result of the deformation (refer to Figure 6: Case 1P: FEA Model: Total Displacement for Thread #1 and the preceding paragraph) that took place. The bending effect and thread sliding that takes place relieves some of the axial displacement on that particular area.



Figure 8: Case 1P: FEA Model: Shank Section Stiffness

Figure 9 shows the bolt and nut thread deflections for Case 1P.



Figure 9: Case 1P: FEA Model: Thread Deflection

Figure 10 shows the calculated thread stiffness for Case 1P. Since the bolt and thread stiffness reported from the FEA Model vary significantly, and each bolt or nut thread in Simplified Structural Model has the same stiffness, it is likely that the models will produce much different results for Case1P.



Figure 10: FEA Model: Thread Stiffness

Recall that rather than using hand calculations, a simple FEA was performed to determine thread stiffness for the Simplified Structural Model. Since the nut and bolt threads are in series, the combined thread stiffness, *keq*, could be approximated using the bolt thread stiffness, *bt* and the nut thread stiffness, *nt*, as follows:

$$keq = \frac{bt * nt}{bt + nt}$$

This calculation would yield a combined thread stiffness of 1.99×10^9 N/m for Cases 1P and 1C and a combined thread stiffness of 1.16×10^9 N/m for Cases 2P and 2C. These values are different than the values shown in Table 5, because the above formula is derived for a situation involving a rigid connection between the two springs. Since the FEA is able to solve this contact problem, the combined thread stiffness obtained from the FEA was used instead for the Simplified Structural Model.

Table-4 summarizes average stiffness calculations reported from the FEA Model for each of the cases. This can be compared with Table-5, which summarizes average stiffness estimates that are calculated for the Simplified Structural Model. Recall that rather than using hand calculations, a simple FEA was performed to determine thread stiffness for the Simplified Structural Model. Since the nut and bolt threads are in series, the combined thread stiffness, *keq*, could be approximated using the bolt thread stiffness, *bt* and the nut thread stiffness, *nt*, as follows:

$$keq = \frac{bt * nt}{bt + nt}$$

This calculation would yield a combined thread stiffness of 1.99×10^9 N/m for Cases 1P and 1C and a combined thread stiffness of 1.16×10^9 N/m for Cases 2P and 2C. These values are different than the values shown in Table 5, because the above formula is derived for a situation involving a rigid connection between the two springs. Since the FEA is able to solve this contact problem, the combined thread stiffness obtained from the FEA was used instead for the Simplified Structural Model.

Case	Bolt Shank	Nut Shank	Bolt Thread	Nut Thread	Combined Thread
	Stiffness [N/m]	Stiffness [N/m]	Stiffness [N/m]	Stiffness [N/m]	Stiffness [N/m]
1 P	7.66x10 ⁹	$7.49 \mathrm{x} 10^{10}$	$1.18 \mathrm{x} 10^9$	$1.16 \mathrm{x} 10^9$	5.68×10^8
1C	6.22×10^9	2.35×10^{10}	1.24×10^{9}	1.09×10^{9}	5.68×10^8
2P	8.46x10 ⁹	$1.90 \mathrm{x} 10^{10}$	1.03×10^{9}	3.87×10^8	2.75×10^8
2 C	6.98x10 ⁹	8.48×10^9	$1.10 \mathrm{x} 10^9$	3.53×10^8	2.64×10^8

Table-5: Simplified Structural Model: Component Stiffness Summary

Case	Bolt Shank	Nut Shank	Bolt Thread	Nut Thread	Combined Thread
	Stiffness [N/m]	Stiffness [N/m]	Stiffness [N/m]	Stiffness [N/m]	Stiffness [N/m]
1P	$7.44 \mathrm{x} 10^9$	3.75×10^{10}	2.96×10^9	$6.09 \mathrm{x10}^9$	$1.60 \mathrm{x} 10^9$
1C	$7.44 \mathrm{x} 10^9$	3.75×10^{10}	2.96×10^{9}	6.09×10^9	$1.60 \mathrm{x} 10^9$
2P	$7.44 \mathrm{x} 10^9$	$1.16 \mathrm{x} 10^{10}$	2.96×10^{9}	$1.90 \mathrm{x} 10^{9}$	9.25×10^8
2 C	$7.44 \mathrm{x} 10^9$	1.16×10^{10}	2.96×10^{9}	$1.90 \mathrm{x} 10^{9}$	9.25×10^8

Table- 6 compares the results from Table-4 and Table-5. A positive percentage difference indicates that the stiffness calculated from the Simplified Structural Model is larger than the stiffness reported from the FEA Model. Bolt and Nut thread stiffness varies significantly between the models because the bending that occurs in the FEA Model is quite significant, and this bending is not accounted for in the Simplified Structural Model.Table-1

Table- 6: Mode	l Stiffness (Compariso	n: Percent	Difference

Case	Bolt Shank	Nut Shank	Bolt Thread	Nut Thread	Combined Thread
	Stiffness Diff.	Stiffness Diff.	Stiffness Diff.	Stiffness Diff.	Stiffness Diff.
1P	-3%	-50%	151%	425%	182%
1C	20%	60%	139%	459%	182%
2 P	-12%	-39%	187%	391%	236%
2 C	7%	37%	169%	438%	250%

Figure 11 to Figure 14 compare the load distributions that were calculated from the FEA Model and the Simplified Structural Model. These results indicate that although the Simplified Structural Model accurately predicts cases with 'Clamped Loading' conditions, the model doesn't accurately predict cases with 'Pulling Type Loading' conditions. This is because of bending; the axial stiffness cannot be accounted for in the Simplified Structural Model. For 'Pulling Type Loading' a larger proportion of the deflection that takes place is radial displacement.

Note how the load distribution approaches zero for the last threads of the clamped loading cases, Case 1C and 2C. This is expected, because the force pulling up on the bolt is equal to the force pushing down on the 'nut'. Therefore, in practical clamped loading situations, if many threads are used in the threaded connection, it is likely that the last threads carry much less of the load as compared to the first threads.

For Pulling Loading Conditions, if the total stiffness of the bolt and the nut were the same, then the first and last threads would carry an equal percentage of the load. This is shown by Case 2P. However, since the nut is much stiffer than the bolt in Case 1P, the load distribution becomes skewed such that the first threads carry a significantly higher percentage of the load as compared to the last threads. The opposite would be true for 'Pulling Type Loading' conditions if the bolt were significantly stiffer than the nut.



Figure 11: Case 1P: Percentage Load Distribution



Figure 12: Case 1C: Percentage Load Distribution



Figure 13: Case 2P: Percentage Load Distribution



Figure 14: Case 2C: Percentage Load Distribution

5. Load Distribution Shifting:

Mechanics report that in some instances when valve covers are taken apart, the (otherwise flat) surface of blocks show raised dimples around tapped holes, a clear sign that upon torquing and operation, the top threads of the female side underwent some plastic deformation. To minimize these effects, counter-boring is used to remove those first threads. This means that the first thread will now have a larger area of material above it, which acts as a 'support shoulder'. This added stiffness reduces deformation of that area of the shank, but when compared to a system that is equal in all other respects (i.e. same number of engaged threads and loading conditions), the load that the first thread carries is sometimes increased significantly. Although a 'support shoulder' has benefits, it is important to be aware that its introduction will cause more load in the first thread.

To understand how to manipulate the load distribution, it is important to understand the physics of a threaded connection, particularly why the load distribution is very rarely uniform. Imagine if the bolt and nut shank sections were several orders of magnitude stiffer than the threads. Since, relatively speaking, the shank sections would be rigid, an equal amount of force would be applied on each of the threads for 'Pulling Type Loading' conditions.

Another way to affect the load distribution is to machine a hole in the shank of the bolt. Typically, industry designers begin drilling this hole through the top of the bolt, and stop before they reach the proximity of the threads. By doing this, they are able to add stretch length to the bolt, which reduces the magnitude of the forces carried by remainder of the threaded connection. However, if a mistake were to be made and the hole were drilled too deep, the load distribution could be negatively influenced. Completely hollowing out the bolt (or shaving off the outer radius of the 'nut') would increase the percentage load that the first threads would experience.

To make the load distribution more uniform in a standard threaded connection is difficult, because increasing the size of the nut and/or bolt not only increases the bolt and nut shank stiffness, but also increases the thread stiffness. Thus, in practice, it is difficult to make the load distribution more uniform by making geometrical changes. However, sometimes making different material choices can help to make the load distribution more uniform. Evidence of this can be found by comparing Figure 13 with Figure 11 and by comparing Figure 14 with Figure 12. As an interesting side note, the reason why in Case 2P the last few threads are carrying almost as much load as the first few threads, is because the total stiffness of the nut and the bolt are nearly identical in this case.

6. Conclusions

- Although the Simplified Structural Model accurately predicts cases with 'Clamped Loading' conditions, the model doesn't accurately predict cases with 'Pulling Type Loading' conditions. This is because bending isn't accounted for in the Simplified Structural Model, and an unacceptably high proportion of the deflection that takes place in cases with 'Pulling Type Loading' conditions is radial displacement.
- 2. Although a 'Support Shoulder' is effective in minimizing the permanent deformation at the top of the 'nut', it can significantly increase the percentage load in the first thread.
- 3. Completely hollowing out the bolt or shaving off material from the outer radius of the 'nut' would make the load distribution less uniform.
- 4. It is difficult to make the load distribution more uniform solely by making geometrical changes, because as the bolt and/or nut size is increased, the stiffness of the threads also increases.
- 5. Making material changes can help to make the load distribution more uniform in some cases.
- 6. In practical clamped loading situations, if many threads are used in the threaded connection, it is likely that the last threads carry much less of the load as compared to the first threads.
- 7. For 'Pulling Type Loading' conditions, if the total stiffness of bolt and the nut were the same, then the first and last threads would carry an equal percentage of the load. If the total stiffness of the nut is higher than the bolt, then the first threads on the threaded connection will carry more of the load than the last threads. If the total stiffness of the bolt is higher than the nut, then the last threads of the threaded connection will carry more the load than the first threads. These two statements are true for all cases that have 'Pulling Type Loading' conditions.

7. References

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